IRSTI 06.73.02 https://doi.org/10.26577/be-2019-4-e1

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MONETARY POLICY OPTIMIZATION BASED ON THE DSGE MODEL OF KAZAKHSTAN'S ECONOMY

Abstract. The model describes the economy in the short term (excluding investments), in the case of inflation targeting policy and represents a system of 15 linearized equations for key macroeconomic indicators of the main economy sectors: households, enterprises, the National Bank, and the external sector. The parameters were estimated by Bayesian methods for the period 2010-2018 and in the sub-period 2015-2018. The advantage of the approach is the possibility of estimating parameters in short time series due to the use of prior information. From the estimates obtained, it follows that the National Bank pays attention not only to inflation, but also to business activity and changes in the exchange rate. As is known from theory, the optimal policy for the monetary regulator may differ from the optimal one for society. To determine the parameters of the optimal monetary policy, the function of social losses was derived and it was shown that, in addition to the traditional variables of the output gap and inflation, the fluctuations in the interest rate and exchange rate should be its components. The work takes into account some sources of welfare losses. These average annual losses of society are estimated at 3.2% of the equilibrium level of consumption. The optimization carried out according to the current version of the DSGE-model allows us to draw the following conclusions. A "double mandate" policy and the inclusion of an exchange rate in Taylor's equation can increase public welfare. The sensitivity coefficients of the current interest rate policy can be revised upward, due to which society losses can be reduced. When pursuing a monetary policy, one should focus not on the CPI, but on indicators of internal inflation, perhaps an indicator of core inflation and/or PPI.

Key words: dynamic stochastic general equilibrium models, Bayesian estimation, inflation targeting.

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Қазақстандағы динамикалық стохастикалық жалпы тепе-теңдік моделі

Аңдатпа. Модель экономиканы қысқа мерзімді перспективада (инвестицияларды қоспаған-да), инфляциялық таргеттеу режимінде сипаттайды және экономиканың негізгі секторларының: үй шаруашылықтары, нақты сектор кәсіпорындары, Ұлттық банк және сыртқы сектордың негізгі макроэкономикалық көрсеткіштері үшін 15 сызықтық теңдеулер жүйесін ұсынады. Параметрлер Байес әдісімен 2010-2018 жж. бағаланды және 2015-2018 ж.ж. Тәсілдің артық-шылығы априори ақпаратын қолдану арқасында қысқа уақыт қатарында параметрлерді бағалау мүмкіндігінде. Алынған бағалаулардан ҚҰБ ақша-кредит саясатын әзірлеу кезінде инфляцияға ғана емес, сонымен қатар іскерлік белсенділік пен айырбастау бағамының өзгеруіне де назар аударады. Теориядан белгілі болғандай, ақша-несиелік реттеушінің оңтайлы саясаты қоғам үшін оңтайлы саясаттан өзгеше болуы мүмкін. Оңтайлы ақша-кредит саясатының параметрлерін анықтау үшін әлеуметтік шығындар функциясы алынды және шығарылған және инфляциялық алшақтықтың дәстүрлі ауыспалыларынан басқа, пайыздық мөлшерлеме мен валюта бағамының ауытқуы оның құрамдас бөлігі болуы керек екендігі көрсетілді. Жұмыста әл-ауқаттың жоғалуының кейбір көздері ескеріледі. Бұл қоғамның орташа жылдық шығындары тұтынудың тепе-теңдік деңгейінің 3,2% деңгейінде бағаланады. DSGE моделінің қолданыстағы нұсқасына сәйкес жүргізілген оңтайландыру келесі қорытынды жасауға мүмкіндік береді. «Қос мандат» саясаты және айырбас бағамын Тейлор теңдеуіне қосу халықтың әл-ауқатын арттыруы мүмкін.

Ағымдағы пайыздық мөлшерлеме саясатының сезімталдық коэффициенттерін жоғары қарай қайта қарауға болады, соның салдарынан қоғам шығындарын азайтуға болады. Ақша-кредит саясатын жүргізу кезінде ТБИ-ге емес, ішкі инфляцияның көрсеткіштеріне, мүмкін, базалық инфляцияның және / немесе ППИ көрсеткіштеріне назар аудару керек.

Түйін сөздер: жалпы тепе-теңдіктің динамикалық стохастикалық үлгілері, Байес бағалаулары, инфляцияның таралуы.

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Оптимизация денежно-кредитной политики на основе DSGE-модели экономики Казахстана

Аннотация. Модель описывает экономику в краткосрочном периоде (без учета инвестиций), в режиме инфляционного таргетирования и представляет собой систему 15 линеаризованных уравнений для ключевых макроэкономических показателей основных секторов экономики: домашних хозяйств, предприятий реального сектора, Национального банка, внешнего сектора. Параметры были оценены байесовскими методами на периоде 2010-2018 гг. и на подпериоде 2015-2018 гг. Преимущество подхода заключается в возможности оценивания параметров на коротких временных рядах за счет использования априорной информации. Из полученных оценок следует, что НБК при выработке денежно-кредитной политики обращает внимание не только на инфляцию, но и на деловую активность и изменения обменного курса. Как известно из теории, оптимальная политика денежного регулятора может отличаться от оптимальной для общества. Для определения параметров оптимальной денежно-кредитной политики выведена функция общественных потерь и показано, что её составляющими, помимо традиционного переменного разрыва выпуска и инфляции, должны быть колебания ставки процента и обменного курса. В работе учтены некоторые источники потерь благосостояния. Эти среднегодовые потери общества оценены в 3.2% от равновесного уровня потребления. Проведённая оптимизация по текущей версии DSGE-модели позволяет сделать следующие выводы. Политика «двойного мандата» и включение в уравнение Тейлора обменного курса могут повысить общественное благосостояние. Коэффициенты чувствительности текущей процентной политики могут быть пересмотрены в сторону повышения, за счет чего можно сократить потери общества. При проведении денежно-кредитной политики следует ориентироваться не на ИПЦ, а на индикаторы внутренней инфляции, может быть, на показатель базовой инфляции и/или ИЦП.

Ключевые слова: динамические стохастические модели общего равновесия, байесовское оценивание, инфляционное таргетирование.

Introduction

The article aims to optimize the parameters of the monetary policy of the National Bank of Kazakhstan on the basis of the Bayesian DSGE-model of the economy of Kazakhstan.

The DSGE model we use has been described in detail in the paper (Shults, 2019). The model consists of aggregated sectors: households and real sector, world and monetary regulator. Households carry out labor activities, save part of their income in interest-bearing assets, and in cash. Real sector enterprises consume household labor and produce for domestic consumption and export. For short-term forecasting purposes, we take fixed assets as exogenous shock. The external sector generates demand for exported goods and creates supply in the form of imported products. Plus, we assume there are no restrictions on the mobility of capital. The National Bank pursues a policy of inflation targeting, managing the base interest rate.

Thus, the model takes into account the labor market, the goods and services market described by employment and wages, prices and GDP. Financial markets are represented by the foreign exchange market, which equilibrium is described by the tenge exchange rate, and the money market, which key feature is the base interest rate.

Literature review

An important advantage of dynamic stochastic general equilibrium models (DSGE models) over econometric modeling is the availability of neo-

Keynesian microfoundations, i.e. behavioral models that describe decision-making by firms and households within rational expectations and market failures. The latter usually include imperfect competition, price inflexibility and asymmetric information. Reliance on microfoundations makes DSGE models free of the Lucas critique (Lucas, 1976: 19-46).

As a rule, the neo-Keynesian DSGE models describe the situation of monopolistic competition, using the Dixit-Stiglitz (Dixit & Stiglitz, 1977) aggregate and modifications of the general equilibrium model by Blanchard-Kiyotaki (Blanchard & Kiyotaki, 1987). These models describe both household consumption and resource consumption in the manufacturing sector in an imperfectly competitive environment. Pricing under inflexible pricing conditions is often modeled in the DSGE literature using the Calvo scheme (Calvo, 1983), suggesting that not all firms are able to set prices according to optimal ones. Following Rotemberg (1982), losses from non-optimal pricing are described by quadratic functions.

Another important feature is the ability to assess social welfare (or social loss). This feature of DSGE-models is emphasized herein – we will try to derive the approximated social loss function (SLF) in the conditions of market failures from the utility function of households. And then, on the basis of SLF, we will optimize the parameters of interest rate policy under conditions of inflation targeting.

There are two approaches to optimizing monetary policy based on maximizing the utility function of society (minimizing social loss). One is the calculation of the recursive utility function $U_t = u_t + \beta U_{t+1}$, where U is the discounted total utility function on the infinite planning horizon, $u_t$is the moment utility function of the household sector, $\beta$is the discounting factor. The problem of using this approach is that u_t is a nonlinear function that depends on level variables (see for example (1)). Most DSGE models are linear with respect to gaps variables. Accordingly, we will rely on an alternative approach by M. Woodford (2003), which is based on a quadratic approximation of the utility function.

Drobyshevsky et al. (2012) noted that the insufficient capacity of financial markets forces developing countries to borrow from abroad. Accordingly, the high dependence on foreign currency loans, especially in the conditions of export-oriented nature of the economy, leads to the need to smooth out fluctuations in the foreign exchange market. The empirical studies by F. Kartaev (2017) confirm this hypothesis – countries pursuing a policy of "hybrid" inflation targeting (i.e. combine inflation targeting with smoothing the volatility of the foreign exchange market), are more efficient in terms of stimulating output.

For DSGE modeling, the statement above means the need to include the SLF components with an exchange rate and an exchange rate variable in the Taylor equation.

Methodology

Modeling the household sector

The utility function with constant relative risk aversion (CRRA) is used to model household behavior. Households maximize expected total discounted utility:

$$U = E\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\sigma}}{1-\sigma} - \Phi\frac{L_t^{1+\varphi}}{1+\varphi} + \Psi\frac{{m_t}^{1-\psi}}{1-\psi}\right)\right] \to \max \quad (1)$$

with budget constraints in deflated terms:

$$(C_t + m_t + d_t + d_t^W - w_t L_t)(1+\pi_t) =$$
$$= m_{t-1} + (1+R_{t-1})d_{t-1} +$$
$$+ d_{t-1}^W \frac{(1+R_{t-1}^W)E_{t-1}[S_t]}{S_{t-1}}$$

where $\beta \in (0;1)$ is discount rate; L_t is labor supply; w_t is real wages; $d_t$and d_t^W are real assets generating interest income in national and foreign currencies; $R_t$and R_t^W are return on assets in national and foreign currencies; $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is inflation rate, and P_t is the consumer prices level; m_t is real cash balances;S_t is nominal exchange rate (in national currency per unit of foreign currency).

First-order conditions are presented in the following form. Demand function for real cash balances is:

$$\Psi {m_t}^{-\psi} = C_t^{-\sigma}\left(\frac{R_t}{1+R_t}\right) \quad (2)$$

Labor supply function (3) is:

$$\Phi L_t^{\varphi} = C_t^{-\sigma} w_t \quad (3)$$

Euler equation for consumption is:

$$\beta\left(\frac{E[C_{t+1}]}{C_t}\right)^{-\sigma} = \frac{1+E[\pi_{t+1}]}{1+R_t} \qquad (4)$$

The optimal structure of consuming domestic $C_{H,t}$ and imported $C_{F,t}$ goods is determined by solving the following problem (Heijdra Ben et al, 2002). Maximize[1] the composite consumption:

$$C_t = \left((1-\delta)^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + \delta^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \to \max \qquad (5)$$

under budget constraint:

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t \qquad (6)$$

Here $P_t$is the consumer basket cost, consisting of domestic and imported goods. $C_{H,t}$and $C_{F,t}$ are consumption of domestic and imported goods at prices $P_{H,t}$and $P_{F,t}$respectively. $\delta \in (0;1)$ is the share of imported goods in consumption, and $\theta > 1$ is a parameter that shows the population tendency to diversify. Moreover, as it will be shown below, the θ parameter can be interpreted as the elasticity of demand at a relative price.

Let us denote the composite consumer price index as:

$$P = \left((1-\delta)P_H^{1-\theta} + \delta P_F^{1-\theta}\right)^{\frac{1}{1-\theta}} \qquad (7)$$

Then the optimal consumption of domestic and imported goods is given by the expressions:

$$\frac{C_{H,t}}{C_t} = (1-\delta)\left(\frac{P_{H,t}}{P_t}\right)^{-\theta} \qquad (8)$$

$$\frac{C_{F,t}}{C_t} = \delta\left(\frac{P_{F,t}}{P_t}\right)^{-\theta} \qquad (9)$$

In turn, the consumption of domestic goods by similar way is decomposed further. Households are expected to consume a continuum of goods produced under monopolistic competition:

$$C_{H,t} = \left(\int_0^1 C_{H,t}(i)^{1-\frac{1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad (10)$$

Then the optimal consumer basket is formed similarly (8), i.e. the demand for the *i*-th product is like: $C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}$.

Modeling the real sector

Derivation of the New-Keynesian Phillips Curve (NKPC) equation for domestic products is based on the articleSchulz & Oshakbayev (2018).

Under conditions of monopolistic competition, the optimal price is set with a markupμ relative to marginal costs: $p_{H,t}^* = \mu + mc_t$, where$p_{H,t}^* \equiv \ln\left(P_{H,t}^*\right)$, mc_t is the logarithm of marginal costs.

But in each time period *t* a certain proportion of firms $\omega \in (0;1)$ are forced to maintain the unchangeable price. Then each firm that is able to set the price, does not choose the price $p_{H,t}^*$ that is optimal at a given time, but some long-term price $\bar{p}_{H,t}$that will minimize the discounted expected loss (taking into account the probability of invariable prices ω)

$$S(\bar{p}_{H,t}) = \sum_{s=0}^{\infty}(\beta\omega)^s E\left[\left(\bar{p}_{H,t} - p_{H,t+s}^*\right)^2\right] \to \min \qquad (11)$$

As a result, the dynamics of prices for domestic goods is described by the equation:

$$\pi_{H,t} \equiv p_{H,t} - p_{H,t-1} = \qquad (12)$$

$$= \kappa \cdot mcr_t + \beta E\left[\pi_{H,t+1}\right],$$

where $mcr_t = \mu + mc_t - p_{H,t}$ is real marginal costs with a premium μ, and $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}$is Calvo parameter, reflecting the price inflexibility.

To model production in the short term, we will use the Cobb-Douglas function:

$$Y = AL^{\alpha} \qquad (13)$$

where Y is the volume of deflated GDP, L is the number of employed, A is the total factor productivity, and $\alpha \in (0;1)$ is the GDP elasticity by labor.

The production function (13) sets the supply-side GDP. On the demand side, GDP is defined as

[1] The solution of the dual problem (minimization of budget expenditures $P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$for a given composite consumptionC_t) provides the same result.

the aggregate demand of different economic sectors:

$$Y_t \equiv AC_t + C_{H,t} + E_t \tag{14}$$

where AC_t is autonomous consumption, consisting of government spending and investment; E_t is the volume of exports.

The DSGE models of small open economy we know assume the so-called international distribution of risks (Gali et al., 2005). It is based on the proposition of full markets, the existence of Arrow's financial assets, and free access to them. In the export model below, we will use the demand model under monopolistic competition, namely the expression (9). Then the demand for domestic exports is described by the equation:

$$E_t = \gamma \left(\frac{P_{H,t}}{S_t P_{W,t}} \right)^{-\vartheta} Y_{W,t} \tag{15}$$

where $Y_{W,t}$ is the world GDP; $P_{W,t}$ is the world prices expressed in foreign currency; ϑ is preferences to diversify the external consumer basket; γ is a scaling factor.

Financial markets

The model of household behavior is used to derive the uncovered interest rate parity (UIP) equation, which balances the return on assets in national and foreign currencies:

$$\frac{1 + R_t}{1 + R_t^W} = \frac{E[S_{t+1}]}{S_t} \tag{16}$$

The equation (16) can be written in logarithms as $s_t = E[s_{t+1}] + (R_t^W - R_t)$, where $s_t = \ln S_t$. Thus, devaluation expectations and interest rate arbitrage can act as exchange rate drivers.

The law of one price assumes that domestic prices for imported goods $P_{F,t}$ are set on the basis of world prices $P_{W,t}$as:

$$P_{F,t} = S_t P_{W,t} \tag{17}$$

Central banks conduct interest rate policy in accordance with the so-called Taylor rule (Taylor, 1993):

$$R_t - \pi_t = r^n + q_\pi(\pi_t - \pi^T) + q_y \tilde{Y}_t, \tag{18}$$

where $\tilde{Y}_t$ is the output gap, the percentage deviation of GDP from its equilibrium state; π^T is the target inflation rate.

Taylor's rule (18) indicates that the real base rate should rise when inflation exceeds its target or/and when the output gap is positive. Taylor's principle states that in order to stabilize the economy, the interest rate response to inflation deviating from the target must be greater than 1 ($q_\pi > 1$).

Since the interest rate cannot change too often and sharply in response to changes in the economic environment, central banks smooth changes in the interest rate (Chernyavsky et al., 2017). In addition, the monetary regulator can intervene in the exchange rate in the foreign exchange market:

$$\begin{aligned} R_t = (1 - \rho_R)\big(r^n + \pi_t + q_\pi(\pi_t - \pi^T) + q_y \tilde{Y}_t \\ + q_{rer} \Delta rer_t\big) + \rho_R R_{t-1}. \end{aligned} \tag{19}$$

This stabilization interest rate policy aims to achieve equilibrium$\tilde{Y}_t = 0$,$\pi_t = \pi^T$,$\bar{R} = r^n + \pi^T$.

Log linear approximation

The derivation of log-linear approximations is presented in (Shults, 2019). Next, we will denote the percentage deviation of the variables from their equilibrium values by "wave". For example, $\tilde{C}_t = \ln\frac{C_t}{\bar{C}}$ is the percentage deviation of household consumption from equilibrium $\bar{C}$.

We use the equation (4) to obtain a dynamic version of the IS equation:

$$\ln\beta - \sigma\big(E[\tilde{C}_{t+1}] - \tilde{C}_t\big) = E[\pi_{t+1}] - R_t \tag{20}$$

So, in the steady state ($\pi_t = \pi^T$ and $\tilde{C}_t = 0$), the natural interest rate r^n must satisfy the condition $r^n = \bar{R} - \pi^T = -\ln\beta$.

The expression for employment (3) is approximated in terms of gaps:

$$\varphi \tilde{L}_t = -\sigma \tilde{C}_t + \widetilde{w}_t \tag{21}$$

The demand for money (2) can be reduced to the form:

$$\widetilde{m}_t = \frac{1}{\psi} + \frac{\sigma}{\psi}\tilde{C}_t - \eta R_t \tag{22}$$

where $\eta = \frac{1}{\psi \bar{R}}$.

The production function (13)is approximated as:

$$\tilde{Y}_t = \tilde{A}_t + \alpha \tilde{L}_t \tag{23}$$

The linearization of the basic macroeconomic identity (14)gives:

$$\tilde{Y}_t = (1 - w_{CH} - w_E)\widetilde{AC}_t + \\ + w_{CH}\tilde{C}_{H,t} + w_E \tilde{E}_t \tag{24}$$

where w_{CH}, w_E are the share of household consumption of domestic goods and exports in GDP.

The real exchange rate $RER = \frac{P_F}{P}$ and terms of trade $Q = \frac{P_F}{P_H}$, recorded in logarithms, take the form $rer_t = p_{W,t} + s_t - p_t$ and $q_t = p_{W,t} + s_t - p_{H,t}$.

The dynamics of inflation for imported goods is given by the equation:

$$\pi_{F,t} = \Delta rer_t + \pi_t \tag{25}$$

The log-linear approximation for the consumer price index (6) is:

$$p_t \approx (1 - \delta)p_{H,t} + \delta p_{F,t} \tag{26}$$

The consumer inflation can be provided as:

$$\pi_t = (1 - \delta)\pi_{H,t} + \delta \cdot \pi_{F,t} \tag{27}$$

The relationship between the real exchange rate and the terms of trade can be approximated by the expression:

$$rer_t = (1 - \delta)q_t \tag{28}$$

Approximation for export (15):

$$\tilde{E}_t = \vartheta q_t + \tilde{Y}_{W,t} \tag{29}$$

The percentage deviations for consumption of domestic (8) and imported (9) goods:

$$\widetilde{C_{H,t}} = \tilde{C}_t + \theta\delta q_t \tag{30}$$

$$\widetilde{C_{F,t}} = \tilde{C}_t - \theta rer_t \tag{31}$$

The UIP equation (16)in terms of the real exchange rate:

$$rer_t = E[rer_{t+1}] + (R_t^W - E[\pi_{t+1}^W]) - \\ -(R_t - E[\pi_{t+1}]) \tag{32}$$

The gap of real marginal cost:

$$\widetilde{mcr}_t = \tilde{w}_t - \tilde{Y}_t + \tilde{L}_t \tag{33}$$

The model also includes the Phillips equation (12) and the Taylor equation (19).

Social loss function

M. Woodford (2003) justified and derived a quadratic approximation of the utility function for a closed economy. In his model, the non-separable utility function is used. Moreover, (Woodford, 2003) contains extensions for cash stocks, consumption inertia, and prices. This unit was included in the popular neo-Keynesian model by Gali (2008).

The basic new Keynesian DSGE model (Gali, 2008) uses the moment utility function $u_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi}$and the Cobb-Douglas production function, similar to (13). Consumption is a consumer basket of a continuum of goods (10). Pricing in case of monopolistic competition is organized according to the Calvo scheme. Then a quadratic approximation of the expected discounted utility function yields the following function:

$$\mathbb{W} = -\frac{1}{2}E\left[\sum_{t=0}^{\infty}\beta^t\left\{\left(\sigma + \frac{\varphi + 1 - \alpha}{2 - \alpha}\right)\tilde{Y}_t^2 + \frac{\varepsilon}{\lambda}\pi_t^2\right\}\right]$$

where $\lambda = \frac{(1-\omega)(1-\beta\omega)}{\omega}\frac{\alpha}{\alpha+(1-\alpha)\varepsilon}$. That is, monetary policy in a closed economy is aimed at minimizing the average social losses (SLF):

$$\mathbb{L} = \frac{1}{2}\left[\left(\sigma + \frac{\varphi + 1 - \alpha}{2 - \alpha}\right)D[\tilde{y}_t] + \frac{\varepsilon}{\lambda}D[\pi_t]\right]$$

where operator $D[\cdot]$is a variance.

So, the welfare loss is associated with monopolization of the economy and price inertia. The first leads to underproduction and higher prices. Price rigidity leads to non-optimal price structure and reduced resource allocation efficiency.

Gali and Monachelli (2005) point out that in an open economy, the monetary regulator has an incentive to influence the terms of trade, which also affects public welfare. In the special case ($\sigma = \theta = \alpha = 1$) and some other constraints, the quadratically approximated social loss function is:

$$\mathbb{L}_{open} = \frac{1-\delta}{2}\left[(1+\varphi)D[\widetilde{y}_t] + \frac{\varepsilon}{\lambda}D\left[\pi_{H,t}\right]\right]$$

The osr function in the Dynare package is designed to minimize quadratic social loss functions by optimizing Taylor rule parameters. To use it, we specify coefficients before variances and covariances of key variables.

The derivation of a linear-quadratic approximation of the utility function for our model is presented in the Appendix. The social loss function for our model has the form:

$$\begin{aligned}\mathbb{L} = \frac{\varepsilon}{\lambda w_C}\pi_{H,t}^2 - a_Y {x_t}^2 - a_{RER}{e_t}^2 - a_R {r_t}^2 + \\ + \frac{\chi_2\chi_4}{w_{CH}^2}\hat{Y}_t rer_t + \chi_3\tilde{Y}_t\frac{R_t}{\overline{R}} - \chi_3\chi_4 rer_t\frac{R_t}{\overline{R}}\end{aligned} \quad (34)$$

where x_t,e_t,r_t are linear transformations over the output gap, the real exchange rate, and the interest rate, respectively.

It follows from the expression (34)that monetary policy should be aimed not only at stabilizing inflation, but also at stabilizing economic activity and the foreign currency market. At the same time, the social loss function includes not all consumer inflation, but only the price index of domestic producers.

In addition, as the Appendix suggests, welfare is affected by stochastic shocks of aggregate factor productivity, autonomous domestic demand and consumption, external demand, inflation, and interest rates. And the optimal monetary policy (Taylor equation parameters) should depend on the intensity of these shocks.

To optimize the coefficients of the Taylor rule, we need to estimate the coefficients of the model. That's where we're proceeding to.

Bayesian estimation

Bayesian methods (DeJong et al., 2011; Mikusheva, 2014) are becoming an increasingly popular way of estimating the DSGE model parameters. This can be partly explained by the fact that not only statistical data is used for estimation, but also prior judgments: the economic theory provisions, expert judgments, the results of previous studies, including foreign ones. As a result, meaningful results can be obtained even on short time series, as prior information fulfills the lack of statistical observations.

Prior knowledge is given as functions of density distribution $f(\theta)$ of unknown parameters θ. Then, based on the available observations, the posterior distribution function is calculated using the Bayes formula:

$$f(\theta|y) = \frac{f(y|\theta)f(\theta)}{f(y)} \propto L(y|\theta)f(\theta),$$

where $f(y)$ is the observation distribution density function;$f(\theta)$ is the a priori parameter distribution function; $f(y|\theta) = L(y|\theta)$ is the likelihood function. To obtain point estimates, the mathematical expectation, median, or a posteriori distribution mode are calculated $f(\theta|y)$.

To estimate the model parameters, for greater adequacy to the real economy, we will introduce several modifications into the model (12), (19)-(32).

In the equation (20), we take into account the desire of households to smooth consumption and add a lag variable:

$$\begin{aligned}\tilde{C}_t = \rho_C\tilde{C}_{t-1} + (1-\rho_C)E\left[\tilde{C}_{t+1}\right] + \\ + \frac{1}{\sigma}(E[\pi_{t+1}] - R_t + r^n) + e_{C,t}\end{aligned} \quad (35)$$

Similarly, instead of (12)we will use a hybrid NKPC (Gali et al., 1999), taking into account the inertia of inflation:

$$\begin{aligned}\pi_{H,t} = \kappa \cdot \widetilde{mc}r_t + \beta E\left[\pi_{H,t+1}\right] + \\ +(1-\beta)\pi_{H,t-1}\end{aligned} \quad (36)$$

We also add inertia to the equation of imported inflation (25):

$$\begin{aligned}\pi_{F,t} = \rho_{\pi F}\pi_{F,t-1} + \\ +(1-\rho_{\pi F})(rer_t - rer_{t-1} + \pi_t)\end{aligned} \quad (37)$$

In order to take into account the possibility of deviation from the floating exchange rate, the foreign exchange market regulation, we add inertia to the UIP equation (32):

$$\begin{aligned}rer_t = (1-\rho_{RER})E[rer_{t+1}] + \\ +\rho_{RER}\big(rer_{t-1} + (R_t^W - E[\pi_{t+1}^W]) \\ - (R_t - E[\pi_{t+1}])\big)\end{aligned} \quad (38)$$

The following statistics from the Committee on Statistics and the National Bank and from the International Monetary Fund (International Financial Statistics Database) was used: GDP, household consumption, exports, consumer price index, producer price index, monetary aggregate M0, US consumer price index.

Table 1– Parameters of a priori distributions and a posteriori estimation of model coefficients

Parameter	Economic sense	Distribution	A priori		A posteriori mean	
			Mean	STD	2010-2018	2015-2018
r^n	Natural interest rate	Gamma	0.005	0.002	0.0049	0.0049
w_{CH}	Share of household consumption of domestic goods in GDP	Beta	0.4059	0.03	0.4423	0.4253
w_E	Share of exports in GDP	Beta	0.3648	0.08	0.0800	0.1445
α	GDP elasticity by labor	Beta	0.1014	0.08	0.0067	0.0049
σ	Inverse of intertemporal substitution of consumption	Gamma	1	0.9	0.5545	0.1016
β	Discount rate	Beta	0.99	0.008	0.9909	0.9890
ϑ	External demand elasticity at prices	Gamma	1.2293	0.9	0.0772	0.0240
θ	Domestic demand elasticity at prices	Gamma	1.0845	0.9	0.0465	0.0150
ψ	Utility elasticity of cash	Gamma	0.2991	0.25	0.7766	0.5014
η	Money demand elasticity at interest rate	Gamma	0.0394	0.03	0.0953	0.1965
φ	Inverse of labor supply elasticity by wage	Gamma	3	2.9	4.5347	4.2610
κ	Calvo parameter for domestic goods	Gamma	0.132	0.13	0.2209	0.2293
$\rho_{\pi F}$	Measure of inflation inertia on imported goods	Beta	0.0203	0.015	0.0006	0.0202
q_π	Measure of the National Bank's commitment to fighting inflation	Gamma	4.0241	1	5.1424	3.5379
q_y	Measure of the National Bank's commitment to stabilizing output	Gamma	0.4683	0.2	0.4972	0.4623
q_{rer}	Measure of the National Bank's commitment to real exchange rate stabilization	Normal	0.1	1	-2.4759	-2.3921
ρ_R	Measure of inertia of the NBK base rate	Beta	0.75	0.15	0.4441	0.5360
ρ_C	Consumption inertia	Beta	0.9368	0.05	0.9325	0.9395
ρ_{RER}	Real exchange rate inertia	Beta	0.1	0.05	0.1003	0.1003
δ	Imported goods share in the consumer basket	Beta	0.2967	0.2	0.1662	0.1878

Seasonality was eliminated by the Census X-12 method in the EViews 8 package. The trend-cyclic component was excluded by the Hodrick-Prescott filter with the standard parameter for quarterly data $\lambda = 1600$.

The parameters of prior distributions were taken from the previous estimations (Shults, 2019) (Table 2). We have deviated in the following cases: $\sigma = 1$; $\varphi = 3$; $\kappa = 0.132$ $\rho_R = 0.75$.

In the expressions above, we have introduced the following sources of shocks:

– Autonomous demand shock $e_{Y,t}$
– Consumer demand shock $e_{C,t}$
– Total factor productivity shock $\tilde{A}_t$
– World price shock $\pi_{W,t}$
– External demand shock $\tilde{Y}_{W,t}$
– The NBK base rate shock $e_{R,t}$
– World interest rate shock R_t^W
– Price shock $e_{\pi,t}$

Table 2 – Parameters of a priori distributions and a posteriori estimation of shock effects features

Parameter	Economic sense	Distribution	A priori		A posteriori mean	
			Mean	STD	2010-2018	2015-2018
ρ_{eY}	Inertia of the autonomous demand shock $e_{Y,t}$	Beta	0.5969	0.2	0.7691	0.8354
ρ_{eC}	Inertia of the consumer demand shock $e_{C,t}$	Beta	0.0777	0.05	0.4653	0.0795
ρ_{A}	Inertia of the total factor productivity shock $\tilde{A}_t$	Beta	0.8248	0.07	0.7931	0.8194
$\rho_{\pi W}$	Inertia of the world price shock $\pi_{W,t}$	Beta	0.6494	0.14	0.7557	0.7272
ρ_{YW}	Inertia of the external demand shock $\tilde{Y}_{W,t}$	Beta	0.1068	0.05	0.1195	0.1128
ρ_{eR}	Inertia of the NBK base rate shock $e_{R,t}$	Beta	0.9565	0.02	0.9587	0.9574
ρ_{RW}	Inertia of the world interest rate shock R_t^W	Beta	0.8983	0.05	0.8867	0.9335
$\rho_{e\pi}$	Inertia of the price shock $e_{\pi,t}$	Beta	0.9	0.04	0.8847	0.8980
σ_{eY}	Standard deviation of the autonomous demand shock $e_{Y,t}$	Inverse Gamma	0.0437	Inf	0.0289	0.0332
σ_{eC}	Standard deviation of the consumer demand shock $e_{C,t}$	Inverse Gamma	0.0141	Inf	2.0271	0.0099
σ_{A}	Standard deviation of the total factor productivity shock $\tilde{A}_t$	Inverse Gamma	0.0126	Inf	0.0093	0.0074
$\sigma_{\pi W}$	Standard deviation of the world price shock $\pi_{W,t}$	Inverse Gamma	0.3348	Inf	0.3136	0.3392
σ_{YW}	Standard deviation of the external demand shock $\tilde{Y}_{W,t}$	Inverse Gamma	0.1932	Inf	0.0757	0.0684
σ_{eR}	Standard deviation of the NBK base rate shock $e_{R,t}$	Inverse Gamma	0.6579	Inf	0.3683	0.3024
σ_{RW}	Standard deviation of the world interest rate shock R_t^W	Inverse Gamma	0.3657	Inf	0.5958	0.6889
$\sigma_{e\pi}$	Standard deviation of the price shock $e_{\pi,t}$	Inverse Gamma	1.0896	Inf	1.2363	1.2927

The estimation was carried out in the Dynare package for Matlab on quarterly data for two periods: from 2010 to 2018 and from 2015 to 2018. The default numerical method, the Sims algorithm, was used. The number of iterations is 100,000. The settings of the numerical methods were selected in such a way that the acceptance ratio was in the range from 0.2 to 0.3 (0.234 is considered optimal). Estimations are given in Tables 1 and 2 in the posterior mean column.

Results and Discussion

Attention is drawn to the low value of the parameter α (elasticity of output by labor) of the production function. This suggests that in the short term, the dynamics of GDP and employment are linked loosely, and indirectly indicates the inertia of the labor market in post-transition economies. This is also indicated by the low value of the labor supply elasticity by wage (high value of φ).

Estimates of the parameters of the Taylor equation differ slightly from the reference parameters given in (Chernyavsky et al., 2017). Estimates for the interest rate elasticity on inflation are above 2.5, and they decreased after 2015. But the interest rate elasticity on the output gap is at the reference level of 0.5. The base rate inertia is lower than 0.75 of (Chernyavsky et al., 2017).

In general, the estimates obtained for the modified model seem to be more adequate in comparison with the previous results (Shults, 2019) and relatively stable in both periods estimated.

These shocks are modeled by first-order autoregressive equations. The parameters of the model shocks are summarized in Table 2.

The parameter εinfluences the social welfare losses significantly, but it is difficult to estimate it (Zaretsky, 2012) and it is not included in the dynamics equations. In (Mukhamediyev, 2013), 6 is chosen as its value. The cash to consumption ratio $\frac{\bar{m}}{\bar{C}}$ will be calibrated based on the data of 2017-2018 at the level of 70%. The consumption share w_C is calibrated at 80%.

With these parameters, the social loss function has the following weights: 19,943 for inflation, 669 for the output gap, about 0 for the real exchange rate and interest rate. Thus, the resulting social loss function is virtually not distinguished from the form used in the DSGE literature only with inflation and output gap. We carried out optimization under different constraints, leaving the interest rate inertia ρ_R unchanged. The results are summarized in Table 3.

Table 3 – Results of optimization of Taylor equation parameters

	(A)	(B)	(C)	(D)	(E)	(F)
q_π	38.8641	5	3.5	39.2839	5	3.5
q_y	28.3621	2	0.5	28.0918	2	2
q_{rer}	-28.0295	-2.9	-2.3			
Social loss $\mathbb{L}$	260,875	264,933	269,187	260,877	264,933	269,095
Loss relative to (A)		1.6%	3.2%	0.0%	1.6%	3.2%

Column (A). The Taylor equation parameters optimization yields too large coefficient values, which means that the interest rate policy will be very volatile. But the following is noteworthy – the coefficient at inflation is only 1.37 times greater than the coefficient at the output gap, and not 5 times, as in the current interest rate policy (Chernyavsky et al., 2017), or 7 times, as in the estimated Taylor equation.

Column (B). We set limits on the range of possible parameters: q_π from 1 to 5, q_y from 0 to 2, q_{rer} from -3 to 0. It is obvious that the conditional optimum is established at the boundaries of the ranges. The social loss, the percentage deviation from the global optimum in case (A) amounted to 1.6% of the implemented set of parameters. The percentages obtained can be interpreted as losses in real household consumption relative to the equilibrium level.

Column (C). With the estimated parameters of the Taylor equation, the social loss increases to 3.2% relative to the global optimum and to 1.6% compared to the implemented option (B).

The argument against including the exchange rate in the Taylor rule is that monetary authorities find it difficult to predict the currency market. Therefore, we further conducted a series of experiments with the classical Taylor rule (without the exchange rate).

Column (D). Since the weight for the real exchange rate in the social loss function is negligible, the results have not changed much from the result (A). Only the coefficient of inflation rose slightly, and decreased in case of the output gap. It seems reasonable, because consumer prices contain exchange rate component due to the exchangerate pass-through effect.

Column (E) shows the optimization results of the implemented Taylor rule (case (B)) without the exchange rate. If the NBK applied the Taylor equation with coefficients 5 and 2, instead of 2.5 and 0.5 (Chernyavsky et al., 2017), the welfare of the society would increase by 4.4% (relative to the SLF level at 2.5 and 0.5). Since the weight of the exchange rate in SLF is negligible, the SLF value has not changed much relative to the case (B).

Column (F) answers the following question: if the NBK does not explicitly take into account the currency factor in the Taylor equation, then should perhaps a dual mandate policy be implemented? In other words, should the monetary regulator aim not only to stabilize inflation, but also to smooth out output gaps? The answer to this question is already in column (D) – the coefficient of the Taylor equation for inflation should be only 1.4 times greater than the coefficient for the output gap, and not 5-7 times greater as it is now.

In addition, we conducted the following experiments. If the Taylor rule does not use the CPI (consumer price index), but the PPI (producer price index) instead, the social loss is reduced many times. And if the Taylor rule does not use the current values of variables, but their advance, the loss on the contrary will grow by almost 3%.

To visualize the results of Taylor rule optimization, we consider several scenario calculations. The impulse response functions presented below show the reaction of the model variables in response to certain disturbances (shocks) in the economy. We will monitor the effects on key welfare variables: real (net of inflation) wages, employment, inflation.

Let us compare the effects of the growing aggregate demand, for example, due to the increase in budget expenditures, under the current (estimates of 2015-2018 in Table 1) and optimized (column (B) in Table 3) interest rate policy (Figure 1). If aggregate demand increases, so does employment, wages, and inflation. In response, the National Bank raises the base interest rate, which stabilizes the economy near equilibrium. But in the case of an optimized Taylor equation, the interest rate rises stronger. As a result, the fluctuations of variables, especially inflation, around the equilibrium are smaller. In other words, a more active monetary policy leads to a more rapid stabilization of the economy, to a smaller dispersion of variables that form social loss.

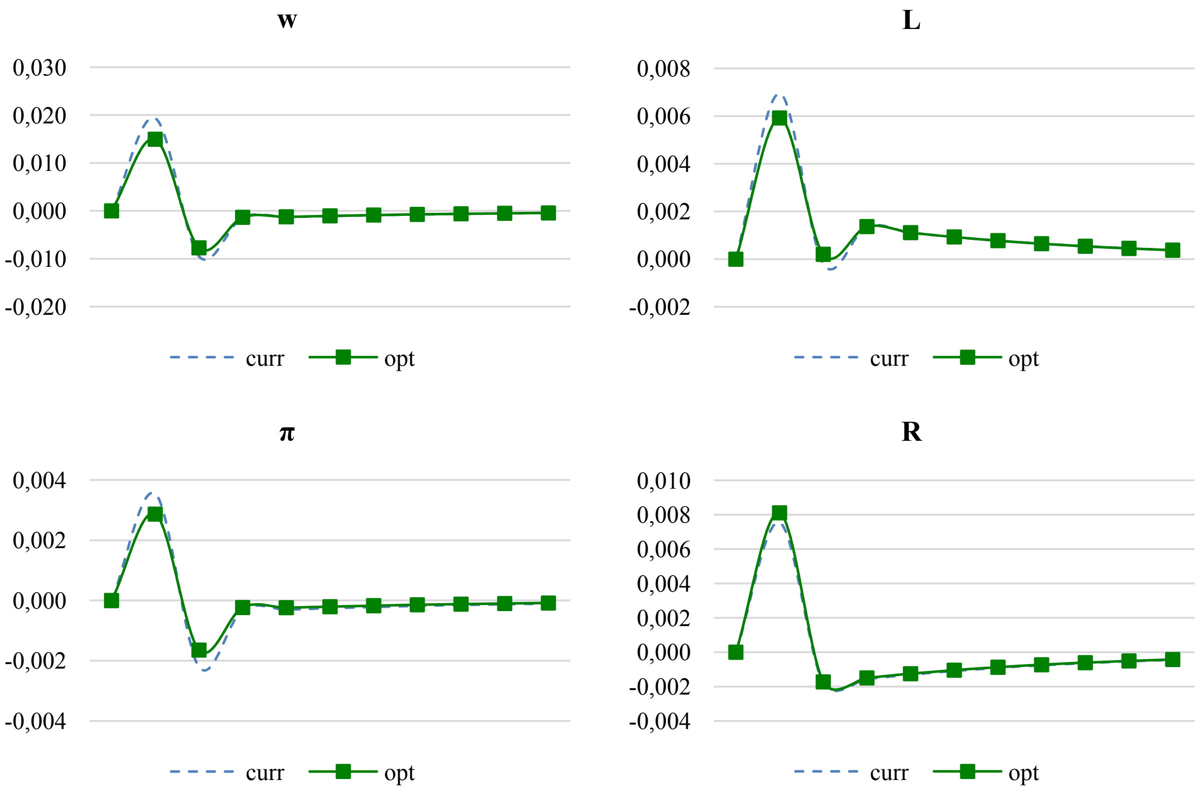

Figure 1 – Response functions under current (curr) and optimized (opt) interest rate policy in case of positive aggregate demand shock

A similar mechanism works in the case of a positive external demand shock (Figure1). The growth of exports leads to employment growth above the natural level and to a subsequent increase in unemployment. Real wages and inflation behave similarly. Accordingly, the monetary regulator is forced to first raise the interest rate, and then reduce it. At the current parameters of the Taylor equation, the amplitude of fluctuations is higher, and the social loss is respectively higher. As mentioned above, the effects of shocks on the economy are smaller at higher values of the Taylor equation coefficients.

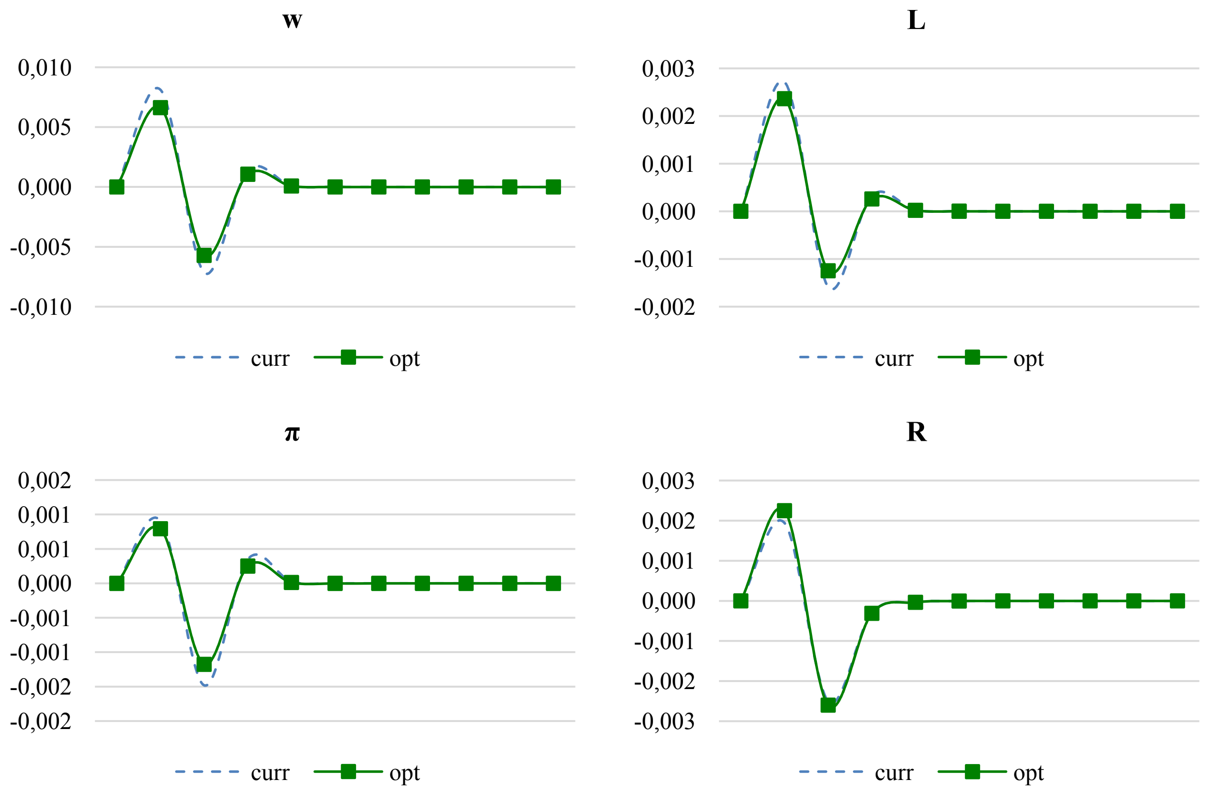

Figure 2 – Response functions under current (curr) and optimized (opt) interest rate policy in case of positive external demand shock

Conclusion

The paper presents the DSGE-model of Kazakhstan. The model parameters are estimated using the Bayesian approach for the period of 2010-2018 and for the subperiod of 2015-2018. The estimates obtained clarify the parameters of the NBK's monetary policy published in (Chernyavsky et al., 2017). In particular, it follows that the NBK, even after the formal transition to the inflation targeting policy, smoothed the fluctuations of the foreign exchange market.

The NBK also seems to pay attention not only to inflation, but also to business activity (dual mandate policy). At the same time, the desire to stabilize inflation decreased after 2015 (3.5 to 0.5), although it is higher than the values indicated in the article (Chernyavsky et al., 2017) (2.5 to 0.5).

It is known that welfare loss can occur primarily due to monopolization of the economy and price rigidity. In addition, as shown in the model, there is a depreciation of income and cash reserves as a result of inflation. And cash reserves lose its value due to the interest rate growth, which acts as an alternative cost of storing money in cash. Also, the decline in employment and consumption leads to welfare loss. These losses are largely due to the volatility of the foreign exchange market.

As a result, the social welfare loss, expressed in units of equilibrium consumption, is 3.2% of the situation of optimal monetary policy.

At the same time, the obtained estimates for welfare loss from currency market fluctuations are "bottom-up" estimates, since our simple model does not take into account many functions performed by foreign currency in the modern economy, foreign trade and the financial system. For example, uncertainty in the foreign exchange market generates financial risks, forcing exporters and importers, as well as the population and banks, to keep a certain reserve of currency to smooth out the effects of exchange rate fluctuations. These buffer stocks of currencies represent a frozen capitaland increase dollarization of economy. Conversely, the capital assets held in the national currency may depreciate in case of an unexpected devaluation. Finally, the conversion of funds into and out of currencies is subject to losses in case of a sharp change in the exchange rate. Thus, it can be concluded that fluctuations in the foreign exchange market increase transaction costs, which reduces the competitiveness of the economy, limits economic growth and employment, and, consequently, reduces social welfare.

Thus, based on the constructed model and carried out optimization, it is possible to draw the following conclusions and recommendations:

– A dual mandate policy and the inclusion of an exchange rate in the Taylor equation can improve social welfare;

– The sensitivity coefficients of the current interest rate policy can be revised upwards, thereby reducing the social loss by half;

– Monetary policy should be guided not by the CPI, but by domestic inflation indicators, perhaps by core inflation.

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Appendix: Quadratic approximation of the utility function

Let us write out the utility function of households (1):

$$U = E\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\sigma}}{1-\sigma} - \Phi\frac{L_t^{1+\varphi}}{1+\varphi} + \Psi\frac{{m_t}^{1-\psi}}{1-\psi}\right)\right]$$

We calculate a quadratic approximation of the utility function $U = \sum_{t=0}^{\infty}\beta^t u_t$related to equilibrium at flexible prices:

$$u_t - \bar{u} \approx \bar{u}_C\bar{C}\left(\frac{C_t - \bar{C}}{\bar{C}}\right) + \bar{u}_L\bar{L}\left(\frac{L_t - \bar{L}}{\bar{L}}\right) + \bar{u}_m\bar{m}\left(\frac{m_t - \bar{m}}{m}\right) + \frac{1}{2}\bar{u}_{CC}\bar{C}^2\left(\frac{C_t - \bar{C}}{\bar{C}}\right)^2 +$$
$$+\frac{1}{2}\bar{u}_{LL}\bar{L}^2\left(\frac{L_t - \bar{L}}{\bar{L}}\right)^2 + \frac{1}{2}\bar{u}_{mm}\bar{m}^2\left(\frac{m_t - \bar{m}}{\bar{m}}\right)^2$$

In the approximation, we took into account the separability of the utility function $\bar{u}_{CL} = 0, \bar{u}_{Cm} = 0, \bar{u}_{mL} = 0$.

Next, we will use the property $Z_t - \bar{Z} \approx \bar{Z}\left(\hat{z}_t + \frac{1}{2}\hat{z}_t^2\right)$. Then:

$$u_t - \bar{u} \approx \bar{u}_C\bar{C}\left(\hat{C}_t + \frac{1}{2}\hat{C}_t^2\right) + \bar{u}_L\bar{L}\left(\hat{L}_t + \frac{1}{2}\hat{L}_t^2\right) + \bar{u}_m\bar{m}\left(\hat{m}_t + \frac{1}{2}\hat{m}_t^2\right) + \frac{1}{2}\bar{u}_{CC}\bar{C}^2\left(\hat{C}_t + \frac{1}{2}\hat{C}_t^2\right)^2 +$$
$$+\frac{1}{2}\bar{u}_{LL}\bar{L}^2\left(\hat{L}_t + \frac{1}{2}\hat{L}_t^2\right)^2 + \frac{1}{2}\bar{u}_{mm}\bar{m}^2\left(\hat{m}_t + \frac{1}{2}\hat{m}_t^2\right)^2$$

These properties are executed for the CRRA utility: $\sigma = -\frac{\bar{u}_{CC}}{\bar{u}_C}\bar{C}$, $\varphi = \frac{\bar{u}_{LL}}{\bar{u}_L}\bar{L}$, $\psi = -\frac{\bar{u}_{mm}}{\bar{u}_m}\bar{m}$. Discarding terms older than the 2nd order, we obtain:

$$u_t - \bar{u} \approx \bar{u}_C\bar{C}\left(\hat{C}_t + \frac{1-\sigma}{2}\hat{C}_t^2\right) + \bar{u}_L\bar{L}\left(\hat{L}_t + \frac{1+\varphi}{2}\hat{L}_t^2\right) + \bar{u}_m\bar{m}\left(\hat{m}_t + \frac{1-\psi}{2}\hat{m}_t^2\right)$$

Under the conditions of monopolistic competition, the percentage deviation of employment from equilibrium at flexible prices is given by the expression $\alpha\hat{L}_t = \hat{Y}_t - a_t + d_t$. Here, the new variable is d_t, the relative price variance (cross-sectional). Under flexible prices $\overline{d_t} = 0$. As proven by Gali (2008), $d_t \approx \frac{\varepsilon}{2\Theta}D[p_{H,t}]$, where $\Theta = \frac{\alpha}{\alpha+(1-\alpha)\varepsilon}$. The parameter $\varepsilon > 0$ reflects the substitution rate between goods in the consumer basket (10).

Considering the above, neglecting terms older than the 2nd order and independent of monetary policy, we obtain:

$$u_t - \bar{u} \approx \bar{u}_C\bar{C}\left(\hat{C}_t + \frac{1-\sigma}{2}\hat{C}_t^2\right) + \frac{\bar{u}_L\bar{L}}{\alpha}\left(\hat{Y}_t + d_t + \frac{1+\varphi}{2\alpha}\left(\hat{Y}_t - a_t\right)^2\right) +$$
$$+\bar{u}_m\bar{m}\left(\hat{m}_t + \frac{1-\psi}{2}\hat{m}_t^2\right)$$

In the next step, we note that under equilibrium, the optimum conditions of the consumer (3)$\bar{w} = -\frac{\bar{u}_L}{\bar{u}_C}$ and firms $\bar{w} = \alpha\frac{\bar{Y}}{\bar{L}}$are met. Accordingly, $-\frac{\bar{u}_L}{\bar{u}_C} = \alpha\frac{\bar{Y}}{\bar{L}}$ and $\frac{\bar{u}_L\bar{L}}{\alpha} = -\bar{u}_C\bar{Y} = -\frac{\bar{u}_C\bar{C}}{w_C}$, where w_C is the share of consumption in GDP under equilibrium.

Besides, from the condition (2): $\bar{u}_m = \bar{u}_C\left(\frac{\bar{R}}{1+\bar{R}}\right)$.Then welfare loss, expressed as a percentage of equilibrium consumption, is equal to:

$$\frac{u_t - \bar{u}}{u_C\bar{C}} \approx \hat{C}_t + \frac{1-\sigma}{2}\hat{C}_t^2 - \frac{1}{w_C}\left(\hat{Y}_t + d_t + \frac{1+\varphi}{2\alpha}\left(\hat{Y}_t - a_t\right)^2\right) + \left(\frac{\bar{R}}{1+\bar{R}}\right)\frac{\bar{m}}{\bar{C}}\left(\hat{m}_t + \frac{1-\psi}{2}\hat{m}_t^2\right)$$

Deviation for money demand (22): $\hat{m}_t = \frac{1}{\psi} + \frac{\sigma}{\psi}\hat{C}_t - \frac{R_t}{\psi\bar{R}}$. Then we get, omitting the fixed terms:

$$\hat{m}_t + \frac{1-\psi}{2}\hat{m}_t^2 \approx \frac{\sigma}{\psi^2}\hat{C}_t + \frac{1-\psi}{2}\frac{\sigma^2}{\psi^2}\hat{C}_t^2 - \frac{R_t}{\psi^2\bar{R}} + \frac{1-\psi}{2}\frac{R_t^2}{\psi^2\bar{R}^2} - \frac{1-\psi}{\psi^2}\sigma\hat{C}_t\frac{R_t}{\bar{R}}$$

Substituting it in the expression for welfare loss, we obtain:

$$\frac{u_t - \bar{u}}{u_C\bar{C}} \approx \chi_1\hat{C}_t + \frac{1}{2}\chi_2\hat{C}_t^2 - \frac{1}{w_C}\left(\hat{Y}_t + d_t + \frac{1+\varphi}{2\alpha}\left(\hat{Y}_t - a_t\right)^2\right) - \chi_m\frac{R_t}{\bar{R}} + \chi_m\frac{1-\psi}{2}\frac{R_t^2}{\bar{R}^2} - \chi_3\frac{\hat{C}_t R_t}{\bar{R}}$$

where $\chi_m = \frac{1}{\psi^2}\left(\frac{\bar{R}}{1+\bar{R}}\right)\frac{\bar{m}}{\bar{C}}$, $\chi_1 = 1 + \chi_m\sigma$, $\chi_2 = 1 - \sigma + \chi_m\sigma^2(1-\psi)$, $\chi_3 = \chi_m(1-\psi)\sigma$

Now we have both the output gap and the consumption gap. In the next step, we move from consumption to output. Given (24) and (28)-(30), they are related by the following relation:

$$\hat{Y}_t = w_{CH}\hat{C}_t + rer_t\frac{w_{CH}\theta\delta + w_E\vartheta}{1-\delta} + e_{Y,t}^{'}$$

Then, getting rid of the consumption variable, we have:

$$\frac{u_t - \bar{u}}{u_C\bar{C}} \approx \hat{Y}_t\left(\frac{\chi_1}{w_{CH}} - \frac{1}{w_C}\right) + \hat{Y}_t^2\left(\frac{\chi_2}{2w_{CH}^2} - \frac{1+\varphi}{2\alpha w_C}\right) - \frac{d_t}{w_C} - \frac{\chi_1\chi_4}{w_{CH}}rer_t + \frac{\chi_2\chi_4^2}{2w_{CH}^2}rer_t^2 - \chi_m\frac{R_t}{\bar{R}} + \chi_m\frac{1-\psi}{2}\frac{R_t^2}{\bar{R}^2} - \frac{\chi_2\chi_4}{w_{CH}^2}\hat{Y}_t rer_t$$
$$-\chi_3\hat{Y}_t\frac{R_t}{\bar{R}} + \chi_3\chi_4 rer_t\frac{R_t}{\bar{R}} + \frac{1+\varphi}{w_C\alpha}\hat{Y}_t a_t - \frac{\chi_2}{w_{CH}^2}\hat{Y}_t e_{Y,t}^{'} + \frac{\chi_2\chi_4}{w_{CH}^2}rer_t e_{Y,t}^{'} + \chi_3 e_{Y,t}^{'}\frac{R_t}{\bar{R}}$$

where $\chi_4 = \frac{w_{CH}\theta\delta + w_E\vartheta}{w_{CH}(1-\delta)}$

Next, we move on to the variables deviating from the steady state. To do this, we need to relate the deviations from the steady state $\tilde{z}_t$ to the deviations from equilibrium at flexible prices $\widehat{z_t}$. As shown in (Gali, 2008), under the performance shock $\tilde{Y}_t = \hat{Y}_t - \frac{1+\varphi}{\sigma\alpha+\varphi+1-\alpha}a_t$. Then, after the transformation, we get (again, discard the components that do not depend on monetary policy):

$$\frac{u_t-\bar{u}}{u_C\bar{C}} \approx \tilde{Y}_t\left(\frac{\chi_1}{w_{CH}}-\frac{1}{w_C}\right)+\tilde{Y}_t^2\left(\frac{\chi_2}{2w_{CH}^2}-\frac{1+\varphi}{2\alpha w_C}\right)-\frac{d_t}{w_C}-\frac{\chi_1\chi_4}{w_{CH}}rer_t+\frac{\chi_2\chi_4^2}{2w_{CH}^2}rer_t^2-\chi_m\frac{R_t}{\bar{R}}+\chi_m\frac{1-\psi}{2}\frac{R_t^2}{\bar{R}^2}-\frac{\chi_2\chi_4}{w_{CH}^2}\tilde{Y}_t rer_t$$
$$-\chi_3\tilde{Y}_t\frac{R_t}{\bar{R}}+\chi_3\chi_4 rer_t\frac{R_t}{\bar{R}}+\left(\frac{1+\varphi}{w_C\alpha}+\chi_5\left(\frac{\chi_2}{w_{CH}^2}-\frac{1+\varphi}{\alpha w_C}\right)\right)\tilde{Y}_t a_t-\frac{\chi_2}{w_{CH}^2}\tilde{Y}_t e'_{Y,t}+\frac{\chi_2\chi_4}{w_{CH}^2}rer_t e'_{Y,t}+\chi_3 e'_{Y,t}\frac{R_t}{\bar{R}}$$
$$-\chi_3\chi_5 a_t\frac{R_t}{\bar{R}}-\frac{\chi_2\chi_4\chi_5}{w_{CH}^2}a_t rer_t$$

Here $\chi_5 = \frac{1+\varphi}{\sigma\alpha+\varphi+1-\alpha}$

Next, we need to collapse the resulting cluttered appearance into a form suitable for using in Dynare. To do this, we use the expression of the form $az_t^2 - 2z_t e_t - 2z_t z^* = a(x_t-\varepsilon_t)^2 + tip$, where$\varepsilon_t = \frac{e_t}{a}$; $x_t = z_t - \frac{z^*}{a}$; *tip*are terms independent from policy.

$$\frac{u_t-\bar{u}}{u_C\bar{C}} \approx a_Y {x_t}^2 + a_{RER}{e_t}^2 + a_R {r_t}^2 - \frac{d_t}{w_C} - \frac{\chi_2\chi_4}{w_{CH}^2}\hat{Y}_t rer_t - \chi_3\tilde{Y}_t\frac{R_t}{\bar{R}} + \chi_3\chi_4 rer_t\frac{R_t}{\bar{R}}$$

where:

$a_Y = \left(\frac{\chi_2}{2w_{CH}^2}-\frac{1+\varphi}{2\alpha w_C}\right)$, $y^* = \frac{1}{2}\left(\frac{\chi_1}{w_{CH}}-\frac{1}{w_C}\right)$, $\varepsilon_{Y,t} = \frac{\left(\frac{1+\varphi}{w_C\alpha}+2\chi_5 a_Y\right)a_t - \frac{\chi_2}{w_{CH}^2}e'_{Y,t}}{a_Y}$, $x_t = \tilde{Y}_t - \frac{y^*}{a_Y} - \varepsilon_{Y,t}$

$a_{RER} = \frac{\chi_2\chi_4^2}{2w_{CH}^2}$, $rer^* = \frac{\chi_1\chi_4}{2w_{CH}}$, $\varepsilon_{RER,t} = \frac{2}{\chi_4}e'_{Y,t} - \frac{\chi_2\chi_4\chi_5}{w_{CH}^2}a_t$, $e_t = rer_t - \frac{w_{CH}}{\chi_4} - \varepsilon_{RER,t}$

$a_R = \chi_m\frac{1-\psi}{2}$, $r^* = \frac{\chi_m}{2}$, $\varepsilon_{R,t} = \chi_3 e'_{Y,t} - \chi_3\chi_5 a_t$, $r_t = \frac{R_t}{\bar{R}} - \frac{1}{1-\psi} - \varepsilon_{R,t}$

Finally, as shown in (Woodford, 2003), $\sum_{t=0}^{\infty}\beta^t D[p_{H,t}] = \frac{\omega}{(1-\omega)(1-\beta\omega)}\sum_{t=0}^{\infty}\beta^t\pi_{H,t}^2$. Then

$$\mathbb{W} = E\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{u_t-\bar{u}}{u_C\bar{C}}\right)\right] == E\left[\sum_{t=0}^{\infty}\beta^t\left\{-\frac{\varepsilon}{\lambda w_C}\pi_{H,t}^2 + a_Y{x_t}^2 + a_{RER}{e_t}^2 + a_R{r_t}^2 - \frac{\chi_2\chi_4}{w_{CH}^2}\hat{Y}_t rer_t - \chi_3\tilde{Y}_t\frac{R_t}{\bar{R}} + \chi_3\chi_4 rer_t\frac{R_t}{\bar{R}}\right\}\right]$$

where $\lambda = 2\frac{\alpha}{\alpha+(1-\alpha)\varepsilon}\frac{(1-\omega)(1-\beta\omega)}{\omega}$.