IRSTI 06.73.55

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OPTIMAL PORTFOLIO SELECTION FOR A DEVELOPMENT BANK

This paper examines the formation and optimization of the investment portfolio of the development bank, which implements the state policy of financing socially significant projects that contribute to the economic growth of the country. This model takes into account the bank's objectives and risk attitude.

We propose a methodology for forming an optimal portfolio based on the Markowitz theory. An important feature of the proposed methodology is that it takes into account differences in priorities of the development bank and commercial banks. In particular, the development bank is less interested in maximizing profits and is more interested in developing products and industries with high value added. The main focus of our methodology is on practical implementation issues arising because of data availability constraints existing for Kazakh companies. With that focus in mind, we model the portfolio optimization problem for the development bank that invests a limited amount of funds in private companies from various sectors of Kazakhstan's economy. To make this example as useful as possible for the practical activities of Kazakhstan financial institutions, we use real yield data for large Kazakhstani companies listed on the Kazakhstan Stock Exchange.

Key words: development bank, investments, impact investments, portfolio theory, optimization model, quadratic programming.

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Даму банкінің оптималды портфелін таңдау

Осы мақалада елдің экономикалық өсуіне ықпал ететін әлеуметтік маңызы бар жобаларды қаржыландырудың мемлекеттік саясатын жүзеге асыратын даму банкінің инвестициялық портфелін қалыптастыру және оңтайландыру қарастырылған. Осы жұмыстың мақсаты даму банкінің оңтайлы портфелін қалыптастыру үшін оның инвестициялық мақсаттарына және жеке қатеріне сәйкес келетін үлгіні әзірлеу болып табылады.

Қазіргі Марковиц теориясының негізінде оңтайлы портфельді қалыптастыру әдістемесі ұсынылған. Оның ерекшелігі, портфельді қалыптастыру кезінде, даму банкінің портфелін оңтайландыруда әлеуетті инвестициялық басымдығы коммерциялық банктің инвестициялық мүдделіліктерінен айырықшылықтары ескеріледі. Атап айтқанда, даму банкі табысты барынша арттыруға қызығушылық танытпай, жоғары қосылған құн мен өндіріс салалары бар өнімдерді дамытуға мүдделі. Әдіснама қазақстандық компаниялар үшін қажетті шектеулі деректерден туындайтын практикалық ойларға негізделген. Атап айтқанда, даму банкінің портфелін оңтайландыру үшін Қазақстан экономикасының әртүрлі секторларынан таңдалған компанияларына шектеулі қаражат көлемін инвестициялау тәсілі қарастырылып отыр. Келтірілген мысал Қазақстандық қаржы институттарының практикалық қызметіне пайдалы болу үшін Қазақстандық қор биржасына берілген ірі компаниялардың табыстылық ақпараттары пайдаланылады.

Түйін сөздер: даму банкі, инвестициялар, әлеуметтік трансформациялық инвестициялар, портфолио теориясы, оңтайландыру моделі, квадраттық бағдарламалау.

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Выбор оптимального портфеля для банка развития

В данной работе рассматриваются вопросы формирования и оптимизации инвестиционного портфеля банка развития, реализующего государственную политику финансирования социально значимых проектов, содействующих экономическому росту страны. Целью данной работы является разработка модели для формирования оптимального портфеля банка развития, соответствующей его целям инвестирования и индивидуальной склонности к риску.

Предлагается методология формирования оптимального портфеля на основе современной теории Марковица. Отличительной чертой является то, что при формировании портфеля учитываются потенциальные различия в оптимизации портфеля банка развития от оптимальных инвестиционнных процедур коммерческого банка, являющиеся результатом различия в инвестиционных приоритетах. В частности, банк развития менее заинтересован в максимизации прибыли и больше заинтересован в развитии продуктов с высокой добавленной стоимостью и отраслей. В методологии основное внимание уделяется практическим соображениям, возникающим вследствии ограниченного объема необходимых данных для Казахстанских компаний. В частности рассматривается подход к оптимизации портфеля для банка развития, который инвестирует ограниченное количество средств в частные компании из различных секторов экономики Казахстана. Для того, чтобы сделать этот пример максимально полезным для практической деятельности Казахстанских финансовых учреждений, используются реальные данные по доходности крупных казахстанских компаний-эмитентов Казахстанской фондовой биржы.

Ключевые слова: банк развития, инвестиции, социально-преобразующие инвестиции, портфельная теория, модель оптимизации, квадратичное программирование.

Introduction

Socially-transforming investments (investments with a high degree of beneficial social impact) are becoming more important and noticeable in recent years. In comparison with other forms of socially responsible investments, the most important feature of this form of investment is the measurement of the social and environmental returns that it generates (Matthews et al., 2015: 19). A socially-transforming investment is aimed at improving the social sphere and at the same time at obtaining financial benefits. The importance of transforming investment was discussed recently at the World Economic Forum in Davos in 2012. Since then multiple organizations have been established worldwide that are engaged in making socially-tarnsforming investment. Among these organizations are The Foundation for Social Entrepreneurs and Big Society Capital in the United Kingdom, the Swiss Investment Fund for Emerging Markets in Switzerland, the Social Stock Exchanges in Brazil, Singapore, South Africa, Portugal, Germany, London. When contemplating this investment direction it is necessary to weigh the pros and cons of socially responsible investments (Arjaliès, 2010: 57). The development banks are the most likely candidates to assume the responsibility for the development of such investment strategies.

In 2005, the Department of Economic and Social Affairs of the United Nations, prepared a report on «Redefining the role of national development banks» (Kovalev, Rumas, 2016: 17). They identified the following five objectives of development banks:

1) the economic development;
2) the innovative development;
3) the social development;
4) the protection of the environment, adaptation to climate change, sustainable development;
5) maintaining financial stability.

In Kazakhstan, these objectives are fulfilled by the joint-stock company the Development Bank of Kazakhstan. The main directions of its activity are:

– the enhancement and efficiency improvements of the state investment activities;

– the development of production infrastructure and manufacturing sector;

– the assistance for attracting of foreign and domestic investments into the national economy of the country.

All this leads to the fundamental mission of the bank, which is to promote the sustainable development of the national economy by investing in the country's non-primary sector.

Literature review

According to Annaev (Annaev, 2010), the «laissez faire» market relations underlying a free (unregulated) market economy can lead to imbalances and elevated financial risks, as witnessed recently during the Great Financial crisis period. More generally, private business may not always be interested or capable of implementing long-term projects that are beneficial for the society.

The development banks are being created around the world to fill these gaps left by the private investment activities. The development bank of Kazakhstan reflects the geopolitical reality of investment in a developing country. They have to take into account the interests of foreign investors who finance their activities primarily for their private gain. In general, this cooperation between foreign investors and the development bank helps to enhance the national infrastructure, to improve the public and corporate governance, to facilitate the development of alternative industries, and thus to enhance the integration into the international economic space.

The influence of development banks on the economy of a country is an active research area. A recent srtude by Ru (Ru, 2018: 275) conducts a detailed analysis regarding the costs and benefits of government loans in the context of Chinese economic development. Ru concludes that the government-financed loans help the state enterprises to expand, but the expansion tends to displace private firms in the same industry. At the same time, he finds that the industrial loans help private firms in the processing industries, while government financed loans in infrastructure spending help all private firms to expand. Overall, Ru notes that the development banks, such as the World Bank, play an important economic role throughout the world.

In general, the success of national development banks is affected by a variety of factors. Most of them can be combined into two large groups (Matyushkin et al., 2016: 14):

1) the factors affecting the mobilization of financial resources;

2) the factors affecting the use of financial resources.

The development of national stock markets plays an important role in creating the opportunities for a transparent capital financing process both by the private investors and by the development bank.

The classical portfolio theory applies quadratic optimization algorithms to solve the portfolio allocation problems (Markowitz, 1952: 77). The Markowitz model laid the foundation for the modern theory of the investment portfolio and became the most influential development in the field of mathematical finance. Nevertheless, the theory has its limitations, particularly in measuring the portfolio risk. In this regard, other portfolio optimization models were developed and other methods of measuring investment risk were advanced. For example, the Markowitz model became the basis of the capital asset pricing model (CAPM) developed by Sharpe (Sharpe, 1964: 425), Litner (Litner, 1966: 13) and Mossin (Mossin, 1966: 768), independently of each other. Multiple new models emerged, such as the portfolio model with a semivariance risk (Kaplan, 1997: 82), the model with mean absolute deviation risk (Konno, 1991: 519), the Value-at-Risk methodology (Jorion, 1996: 47), the Conditional Value-at-Risk (Mansini et al., 2007: 227), as well as the models constructed on the basis of three key parameters: such as the mathematical expectation, the variance and the conditional cost measure of risk (Najafi et al., 2015: 445), a methodology for determining the portfolio risk based on the underidentified sets theory (Huang, 2008: 351), applicable under conditions of ambiguity (Huang, 2011: 71).

In real world portfolio formulation problems, investors do not limit their attention to just expected profitability and risks. It is necessary to take into account such factors as the type of investment portfolio (Zybin, 2014: 1), the required level of portfolio return (Fisher et al., 2017: 127), the permissible degree of risk (Corter et al., 2006: 369), the degree of portfolio diversification (Yu et al., 2017: 467), the liquidity requirements (Weber et al., 2013: 69), the taxation of income (Turvey et al., 2013: 93), the transaction costs with various types of assets (Paç et al., 2018: 223), etc. Therefore, the investment portfolio models that take into account these additional criteria are becoming more and more popular. Para et al. (Parra et al., 2001: 287) proposed a model that is based on three criteria: profitability, risk, and liquidity. The same criteria are used in the work of Fahn et al. (Fang et al., 2006: 879), based on the theory of underidentified sets.

This study relies both on the original Markowitz model, as well as on the modified optimization methods, but in a different context. When solving the problem of portfolio allocation for the development bank, it is neccessary to take into account that the development bank has more broad investment objectives than a private investor. In particular, the development bank is less interested in maximizing profits and is more interested in developing industries with high added value (Shukaev et al., 2018: 146).

The development bank will pay less attention to net profit (after taxes), but more closely evaluate the total amount of revenue generated by this company, regardless of who receives it: employees as income from wages, lenders in as interest earnings, shareholders as dividends or the state in the form of taxes. It is clear, however, that the size of net profits generated by the companies receiving investment funds, cannot be ignored completely, as the healthy return on shareholders' equity is crucial for the survival of the business units. The importance of this factor is confirmed in the work of Jain (Jain, 1989: 100), based on an analysis of the activities of the Industrial Finance Corporation of India.

According to Bernstein (Bernstein, 2012), the success of an investor in the stock market lies in the systematic approach to funds allocation among the broad categories of assets. Aybazova (Aybazova, 2016: 36) points out that the formation of the company's strategic plan begins with the development of the principles of portfolio management. These principles help to identify promising portfolio projects and other initiatives, and then to select the most attractive projects for inclusion into the strategic portfolio. Considering this, the development bank manager should carry out a portfolio analysis after collecting data on the social return and profitability of investment projects implemented over the past few years by various companies.

Computer simulation algorithms, similar to that described in Shukaev et al. (Shukaev et al., 2016: 76), can be employed for scenario analysis undertaken during this process. The nature of the optimal portfolio analysis will be similar for both private investors, such as commercial banks, and for the development bank. The only significant difference is that to assess the return on investment, the private bank will use the net profit on the shares of companies. On the contrary, the development bank will use the expected social return on investment in choosing its optimal portfolio. In addition, the development bank will have to guarantee an acceptable monetary return from its portfolio and acceptable levels of default risk, to avoid private enterprises that may not be able to pay on loans. To achieve these goals, the development bank needs to compare the optimal portfolios generated by social incomes to those generated by private incomes.

The main institutions of the securities market of Kazakhstan, providing the required level of technical infrastructure are «Kazakhstan Stock Exchange» JSC (KASE), as well as professional market participants. The infrastructure of the stock market is now fully devoped (Niyazbekova, 2014) in line with the similar structures in other Emerging market economies.

In our analysis, we use the actual data on the profitability of Kazakhstan companies, collected from newsletters distributed by Halyk Finance Research through electronic resources of KASE.

The portolio problem of a development bank

This work analyzes a portfolio optimization problem for a development bank that is investing a limited amount of funds into private companies from various sectors of Kazakhstan Economy. Three important considerations affected our portfolio modeling choices.

– First, the investment objectives of the development bank are likely to be different from the objectives of a private bank that is driven only by the profit maximization motives. This note highlights potential differences in the optimal portfolio strategies of the development bank from the optimal portfolio procedures of a commercial bank, created by the differences in investment priorities.

– Second, in order to make this example as practical as possible, this note uses the actual balance sheets and investment return data for large Kazakhstan companies listed on the Kazakhstan Stock Exchange (KASE).

– Third, since the note is making a methodological contribution to the literature on optimal portfolio choice of a development bank, we devote substantial attention to practical considerations and limitations imposed by the data availability issues.

The development bank has to guarantee acceptable monetary returns on its portfolio and acceptable levels of default risk to avoid unviable private enterprises, which are likely to default on loans. In order to achieve these goals, the development bank has to compare the optimal portfolios generated by social returns to those generated by private returns. For these reasons, the subsequent analysis focuses on a portfolio optimization problem of a private bank, using the actual investment return data collected from the newsletters distributed by Halyk Finance Research service.

We collected the return and market cap series from Halyk Finance Daily letters received between September 5, 2013, and May 31, 2017. After removing incomplete series, we obtained the monthly return and market cap series on 11 companies from four industries: 1) the infrastructure; 2) the metals production; 3) the oil extraction; and 4) the telecommunication industry. We use the following three variables in our analysis:

1. The expected return over the 12 month period following the last observation day on May 31, 2017. These numbers represent Halyk Bank Research's forecasts of expected total returns, over the next 12 months, from holding shares of each company. The expected total return includes both the expected share price appreciation and the expected dividend payments. Table 1 shows these expected total return numbers for all 11 companies in our sample.

Table 1 – Expected 12 month total returns, as of May 31, 2017

Industry	INFR	INFR	Metals	Metals	Metals	Oil	Oil	Oil	Tele-com	Tele-com	Tele-com
Company	KZTO	KEGC KZ	CAML LN	GB KZMS	KAZ LN	KMG LI	NOG LN	RDGZ	Kcel LI	Kcel KZ	KZTK
Exp.12 M Return	29%	-30%	38%	27%	29%	14%	9%	29%	23%	15%	9%

These numbers represent the expected total return on equity for the next 12 months, forecasted by Halyk Bank Research. The spread of interest rates in the presented table does not reflect absolute risk levels. As practice shows, the yield of securities can be both positive and negative in different periods of ownership.

The 12-month forecast horizon is relatively short for long-term investors, including development banks. It would be preferable to have longer-term forecasts, such as, for example, expected returns for the next 5-10 years. Economists and specialists of financial markets working in private banks or development banks formulate long-term forecasts based on statistical analysis of historical data, computer simulation of financial models, or through professional forecasts.

2. The monthly total returns series for each company and for each month between March 2015 and May 2017. These numbers represent realized monthly returns from holding the shares of the 11 companies we selected. Table 2 shows a substantial variation in both the means and the standard deviations of these monthly return series across the companies in our sample.

Table 2 – Average monthly returns and their standard deviations

Industry	Infr	Infr	Met	Met	Met	Oil	Oil	Oil	Tel	Tel	Tel
Com-pany	KZTO	KEGC KZ	CAML LN	GB KZMS	KAZ LN	KMG LI	NOG LN	RDGZ	Kcel LI	Kcel KZ	KZTK
Average	2.36	2.58	1.33	5.98	6.19	0.59	0.55	1.00	-2.61	0.44	1.64
St. dev.	8.70	10.02	8.06	19.56	23.20	10.07	12.26	9.58	13.73	10.17	9.82

In the optimal portfolio analysis we will need not just the standard deviations, shown in the last row of Table 2, but also the full variance-covariance matrix of monthly returns. Table 3 shows the variance covariance matrix of monthly returns. Note that the numbers on the main diagonal of the variance-covariance matrix are just variances of individual returns, equal to the squared standard deviation numbers from Table 2.

The market capitalization series for each company and each month between March 2015 and

May 2017. The market cap numbers represent the total value of all outstanding common shares of each company, expressed in U.S. dollars. The market cap data can be used to calculate market cap weighted average sectoral returns, as well as the market cap weighted average return on a portfolio including all 11 companies in our sample. Table 4 partially shows the market cap data in our sample.

Table 3 – Variance-Covariance matrix of monthly returns

	A	B	C	D	E	F	G	H	I	J	K
1	INFR	INFR	Metals	Metals	Metals	Oil	Oil	Oil	Telecom	Telecom	Telecom
2	KZTO	KEGC KZ	CAML LN	GB_KZMS	KAZ LN	KMG LI	NOG LN	RDGZ	Kcel LI	Kcel KZ	KZTK
3	75.76	44.85	27.95	101.51	116.82	51.34	21.24	13.29	49.71	24.94	27.28
4	44.85	100.40	6.27	40.43	24.26	34.63	-14.04	22.55	-3.01	8.01	37.27
5	27.95	6.27	65.01	68.60	72.82	28.79	36.87	2.26	14.42	11.56	-0.25
6	101.51	40.43	68.60	382.58	411.21	65.77	44.08	4.56	0.72	-10.45	-19.25
7	116.82	24.26	72.82	411.21	538.07	107.75	68.67	10.00	6.74	-41.46	-33.81
8	51.34	34.63	28.79	65.77	107.75	101.42	28.51	69.79	64.48	17.85	22.36
9	21.24	-14.04	36.87	44.08	68.67	28.51	150.38	-10.64	28.50	2.53	17.83
10	13.29	22.55	2.26	4.56	10.00	69.79	-10.64	91.78	44.37	21.63	3.41
11	49.71	-3.01	14.42	0.72	6.74	64.48	28.50	44.37	188.50	112.72	65.05
12	24.94	8.01	11.56	-10.45	-41.46	17.85	2.53	21.63	112.72	103.42	39.50
13	27.28	37.27	-0.25	-19.25	-33.81	22.36	17.83	3.41	65.05	39.50	96.46

Table 4 – Market cap data in millions of U.S. dollars

	A	B	C	D	E	F	G	H	I	J	K	L
1		INFR	INFR	Metals	Metals	Metals	Oil	Oil	Oil	Telecom	Telecom	Telecom
2	Dates	KZTO	KEGC KZ	CAML LN	GB_KZMS	KAZ LN	KMG LI	NOG LN	RDGZ	Kcel LI	Kcel KZ	KZTK
3	May 31 2017	1564	1106	315	2943	2895	3977	1193	4021	660	726	540
4	April 28 2017	1487	1081	338	2830	2836	4032	1068	4038	698	716	560
5	March 31 2017	1422	1110	298	2650	2593	4159	1102	4260	720	738	597
6	February 28 2017	1463	1087	342	2936	2911	4283	1143	4168	730	747	573
7	January 31 2017	1450	1073	330	2616	2437	4003	1022	3877	719	693	477
8	December 30 2016	1260	897	308	2087	1964	3095	865	3239	640	641	463
9	November 30 2016	1196	838	297	2096	2033	3213	981	3264	640	652	440
10	October 31 2016	1244	841	256	1652	1609	3148	788	3167	710	695	441

With the data provided we can proceed to the optimal portfolio analysis examples. We start with the simplest example first.

Let us consider the problem for an investor with a fixed investment budget that has to be fully allocated between the shares of the N companies (in our example $N = 11$). Thus the investor cannot borrow additional funds from other entities and is not allowed to invest into any other assets outside of the given set of 11 companies. We will also ignore for now any diversification or sectoral composition requirements. Furthermore, in all of our examples below, we assume that the investor can only buy a non-negative amount of shares for each of the 11 companies. The non-negativity constraint on the shareholdings implies that the investor is prohibited from taking short positions against individual companies.

The non-shorting constraint accords very well with the actual practice of development banks, which are typically not expected to short any companies for ethical or risk management reasons. The full allocation of all earmarked investment funds is also desired, but is not a necessary requirement. Later we will impose some industry diversification restrictions since such restrictions are typically expected in the development bank investment practice.

Example 1. Optimal portfolio allocations without shorting, with full allocation of funds, and no diversification restrictions.

The investor has to allocate the total amount S of funds, between N companies which will remain fully invested for the next year. The optimal portfolio problem is formulated as follows:

$$\min_{\{X_i\}_{i=1}^N}\left[\operatorname{var}\left(\sum_{i=1}^{N} X_i R_i\right)\right] \qquad (1)$$

subject to the constraints:

$$\sum_{i=1}^{N}\left(X_i R_i^E\right) \geq S \times \overline{R} \qquad (2)$$

$$\sum_{i=1}^{N} X_i = S \qquad (3)$$

$$X_i \geq 0, \text{ for all } i = 1,2,...,N,\ N = 11. \qquad (4)$$

The decision variables X_i are the amounts of money invested into each of the N companies. The non-negativity constraint (4) and the full allocation constraint (3) imply that each X_i has to be between zero and S.

Both R_i^E and R_i represent net Tenge returns resulting from a 100 Tenge investment into the shares of company i. The superscript E on the return values R_i^E stands for the expected return value and accounts for the uncertainty regarding the realized returns R_i from each company $i = 1,2,\ldots,N$ over the subsequent 12 month period. We will be using the Exp.12MReturn data from Table 1 as our expected R_i^E values for each company $i = 1,2,\ldots,N$. For example, $R_1^E = 0.29$ represent expected net return for KZTO, meaning that every 100 Tenge invested into KZTO is expected to generate 29 Tenge of net return over the subsequent 12 month period.

The variables R_i in the variance term of (1) stand for the actual realization of the net return after a 12 month period. Since the actual return realizations R_i are subject to uncertainty, the problem minimizes the expected variance of the portfolio returns. It is important to understand the relevance of the variance-covariance matrix from Table 3 in the general formulation of the problem (1)-(4). We can decompose the variance of the portfolio returns as the weighted sum of the variances and covariances of individual assets entering the portfolio:

$$\operatorname{var}\left(\sum_{i=1}^{N} X_i R_i\right) = \sum_{i=1}^{N} X_i^2 \operatorname{var}\left(R_i\right) + 2\sum_{i=1}^{N}\sum_{j>i}^{N} X_i X_j \operatorname{cov}\left(R_i, R_j\right), \qquad (5)$$

where each $\operatorname{var}(R_i)$ in the first term on the right of equation (5) is approximated by the numbers on the main diagonal of Table 3 (i.e. 75.76, 100.40, 65.01, …, 96.46), while each pairwise covariance term $\operatorname{cov}(R_i, R_j)$ is approximated by the corresponding entries above the main diagonal of Table 3. For example, $\operatorname{cov}(R_1, R_2) = 44.85$ and $\operatorname{cov}(R_{10}, R_{11}) = 39.50$.

The non-negative parameter $\overline{R}$ in the constraint (2) determines the expected return the investor is trying to attain. Typically, the higher is the parameter $\overline{R}$ the higher is the portfolio risk that the investor has to accept in its portfolio choices. For example, if $\overline{R} = 0.38$ corresponding to the highest expected return from Table 1 of 38 percent, the investor will have to invest his or her entire portfolio into just one company, CAML LN. This investment strategy will be risky of course, due to lack of diversification. For lower values of $\overline{R}$ the optimal portfolio is likely to include other assets, with lower expected portfolio return, but also lower portfolio risk. The values of $\overline{R} > 0.38$ are not justified with the given set of companies and expected returns.

Note that it is easy to state the problem (1)-(4) in a scale-free way. To see that clearly, reformulate the problem (1)-(3) in terms of investment shares $x_i = X_i / S$, for all $i = 1,2,\ldots,N$. The portfolio shares x_i must sum up to one. The equivalent reformulated problem looks as follows:

$$\min_{\{x_i\}_{i=1}^N} \operatorname{var}\left(\sum_{i=1}^{N} x_i R_i\right) \qquad (6)$$

subject to the constraints:

$$\sum_{i=1}^{N}\left(x_i R_i^E\right) \geq \overline{R} \qquad (7)$$

$$\sum_{i=1}^{N} x_i = 1 \qquad (8)$$

$$x_i \geq 0 \text{ for all } i = 1,2,...,N,\ N = 11. \qquad (9)$$

Thus, without loss of generality, we can normalize $S = 1$ from now on.

The problem (6)-(9) can be solved using quadratic programming algorithms. Solving the problem (6)-(9) for various values of $\overline{R}$ we can visualize the set of optimal solutions as the implied efficient risk-return trade off in the return-volatility space, see Figure 1.

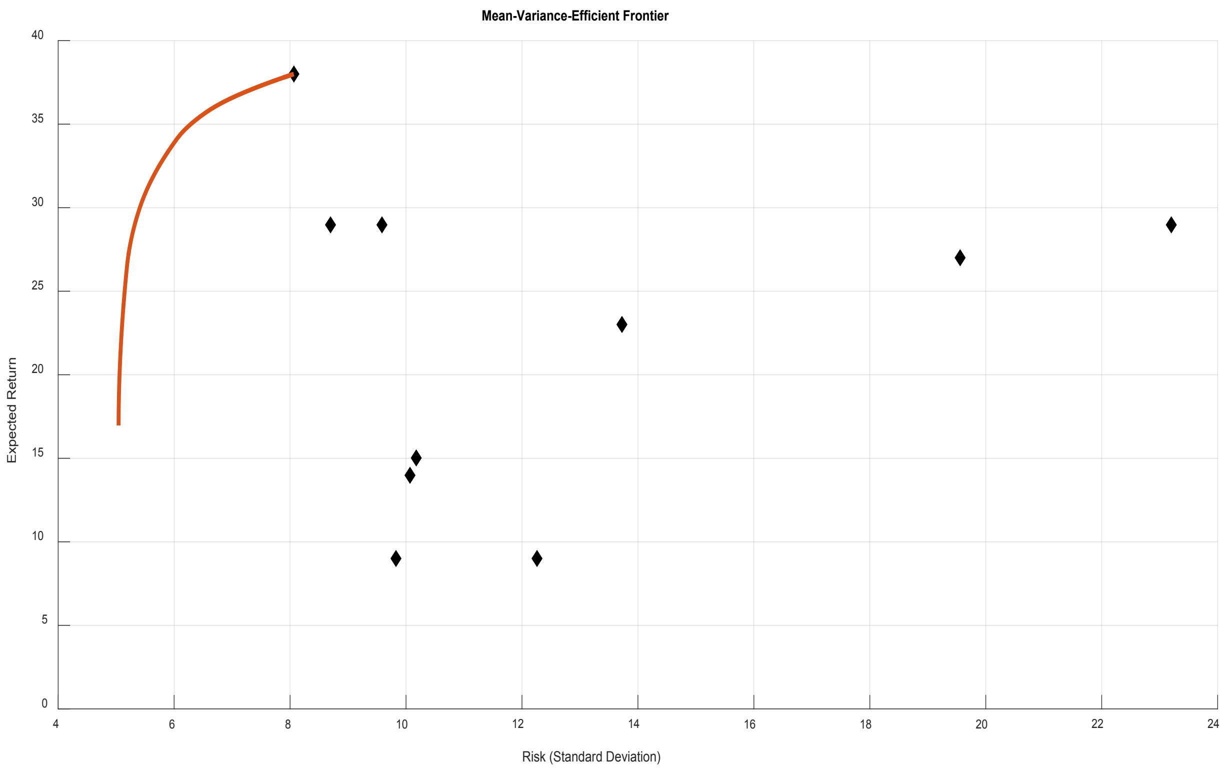

Figure 1 – The risk return trade off of the optimal portfolio

The black diamonds on Figure 1 represent the expected 12 month return and the expected standard deviation of returns for individual companies. The red curve shows the expected 12 month return and the expected standard deviations of returns for the optimal portfolios generated with various values of the parameters $\bar{R}$. The higher is $\bar{R}$ the higher is the optimal portfolio on the red curve. Note that the efficient portfolios have much lower volatilities than most individual company shares. This is possible because the optimal portfolio exploits the cross-correlation between individual assets to lower the aggregate portfolio risk.

Let us now modify the optimal portfolio problem to introduce a risk-free alternative asset, which can be used to reduce the risk of the portfolio even further.

Example 2. Optimal portfolio allocations without shorting and no diversification restrictions, but with an additional risk-free asset.

As before the investor has to allocate the total amount $S=1$ of funds, but besides the 11 aforementioned companies, it also has the option of investing a fraction $x_0 \in [0,1]$ of funds into risk-free government bonds. The government bonds guarantee the net return R_0 after 12 months. In our numerical computations we will assume that the rate of return on government bonds is given by the National Bank of Kazakhstan Base rate, which is currently set at 10.5 percent. The optimal portfolio problem is formulated just like before, with the only difference that the index i can now also be equal to zero.

$$\min_{\{x_i\}_{i=0}^{N}} \operatorname{var}\left(\sum_{i=0}^{N} x_i R_i\right) \tag{10}$$

subject to the constraints:

$$\sum_{i=0}^{N} x_i R_i^E \geq \bar{R} \tag{11}$$

$$\sum_{i=0}^{N} (x_i) = 1 \tag{12}$$

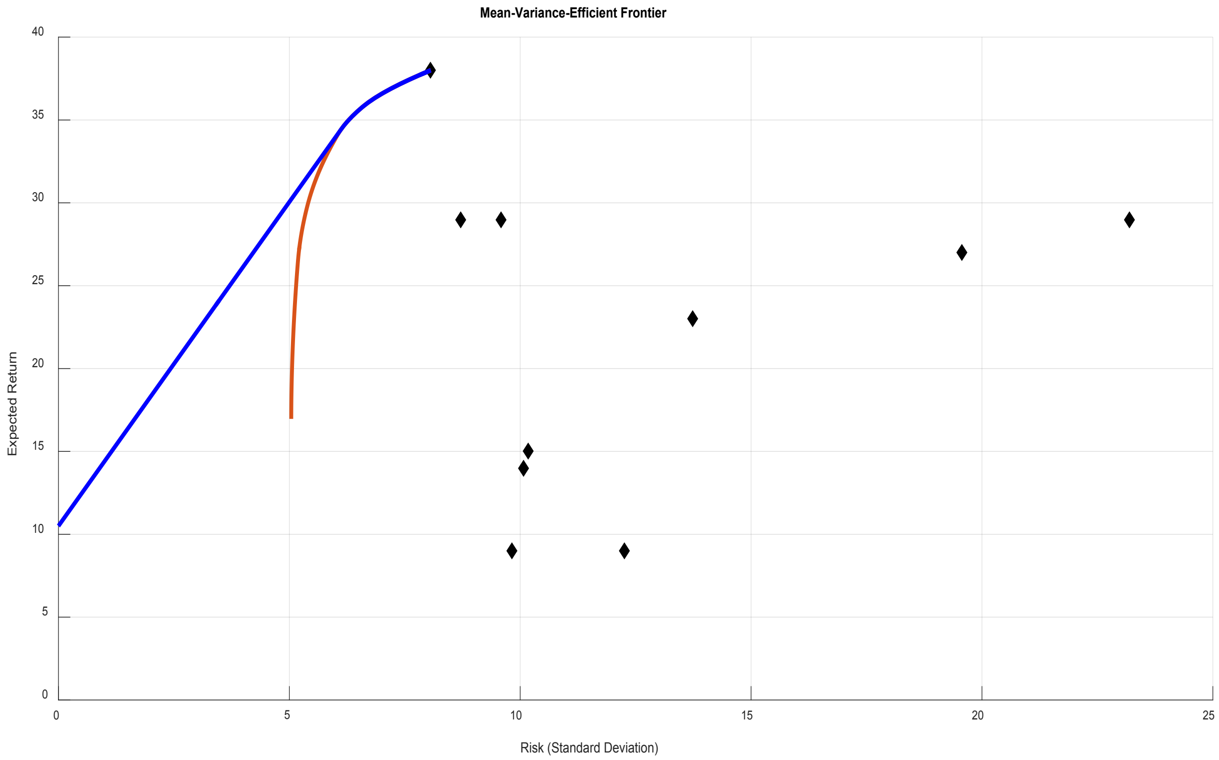

Figure 2 – The risk return trade off of the optimal portfolio with bonds

$$x_i \geq 0, \text{ for all } i = 0,1,2,...,N, \ N = 11. \quad (13)$$

Since the government bond's return is assumed to be risk-free, its expected variance and its covariances with other assets are assumed to be equal to zero. Once we add the government bond to the portfolio choice problem and solve it again for various values of the parameter $\overline{R}$, we obtain the new efficient risk-return frontier shown by the blue curve in Figure 2. Naturally, the risk-free bonds allow reducing the portfolio risk relative to the frontier without risk-free bonds, shown by the red curve in Figure 2.

Example 3. Optimal portfolio allocation without shorting, but with sectoral diversification restrictions and with the risk-free asset

In practice fund managers rarely develop risky portfolios without imposing some diversification restrictions. For example it is typically unwise to invest more than 20 percent of total portfolio value into one company's shares. Moreover fund managers often face some industry composition restrictions. For example, a fund manager might be unwilling to invest more than 20 percent of total portfolio into resource extracting industries, because the returns in such industries tend to be highly volatile due to commodity price fluctuations. [1] In addition, the resource extracting industries might be less appealing targets for investment from the development bank's point of view, because they tend to be well funded by private investors, and have a lower social rate of return than other industries in Kazakhstan.

In this example we impose both types of constraints. First, we assume that none of the assets other than government bonds should receive more than 20 per cent of total investment portfolio. Government bond investments remain unrestricted. Second, we assume that the Metal and Oil industries together should not account for more than 20 percent of total portfolio. These diversification restrictions on the composition of assets modify our optimization problem in the following way:

[1] Using the Market cap data in Table 4 as relative weights, we constructed the sectoral monthly returns and then computed their standard deviations. The standard deviations of sectoral returns are: 8.7% in Infrastructure, 18.6% in Metals, 8.0% in Oil and 10.1% in Telecommunications. Thus the sectoral returns in Metals industry indeed appear much more volatile than in other industries, including Oil.

$$\min_{\{x_i\}_{i=0}^{N}} \operatorname{var}\left(\sum_{i=0}^{N} x_i R_i\right) \qquad (14)$$

subject to the constraints:

$$\sum_{i=0}^{N} x_i R_i^E \geq \overline{R} \qquad (15)$$

$$\sum_{i=0}^{N} x_i = 1 \qquad (16)$$

$$x_i \geq 0, \text{ for all } i = 0,1,2,...,N, \; N = 11. \qquad (17)$$

$$x_i \leq \overline{x}_i, \text{ for } i = 1,2,...,N, \; N = 11, \; \overline{x}_{i=1,...,N} = 0.2. \qquad (18)$$

$$\sum_{x_{i \subset Ind}} x_i \leq 0.2, \; Ind = \{3,4,5,6,7,8\}. \qquad (19)$$

where the last constraint sums the fractions corresponding to all companies from the Metal and Oil industries, in the order they are presented in the Tables 1-4.

Solving the portfolio choice problem with the additional restrictions (17) and (18), for various values of the expected return parameter $\overline{R}$, we obtain the new efficient risk-return frontier shown by the purple curve in Figure 3. Notice how the diversification restrictions lower the portfolio returns by bending the upper branch of the efficient frontier to the right relative to the unrestricted optimal frontiers.

Once the efficient frontier portfolios are identified, the investor can proceed with the allocation of funds.

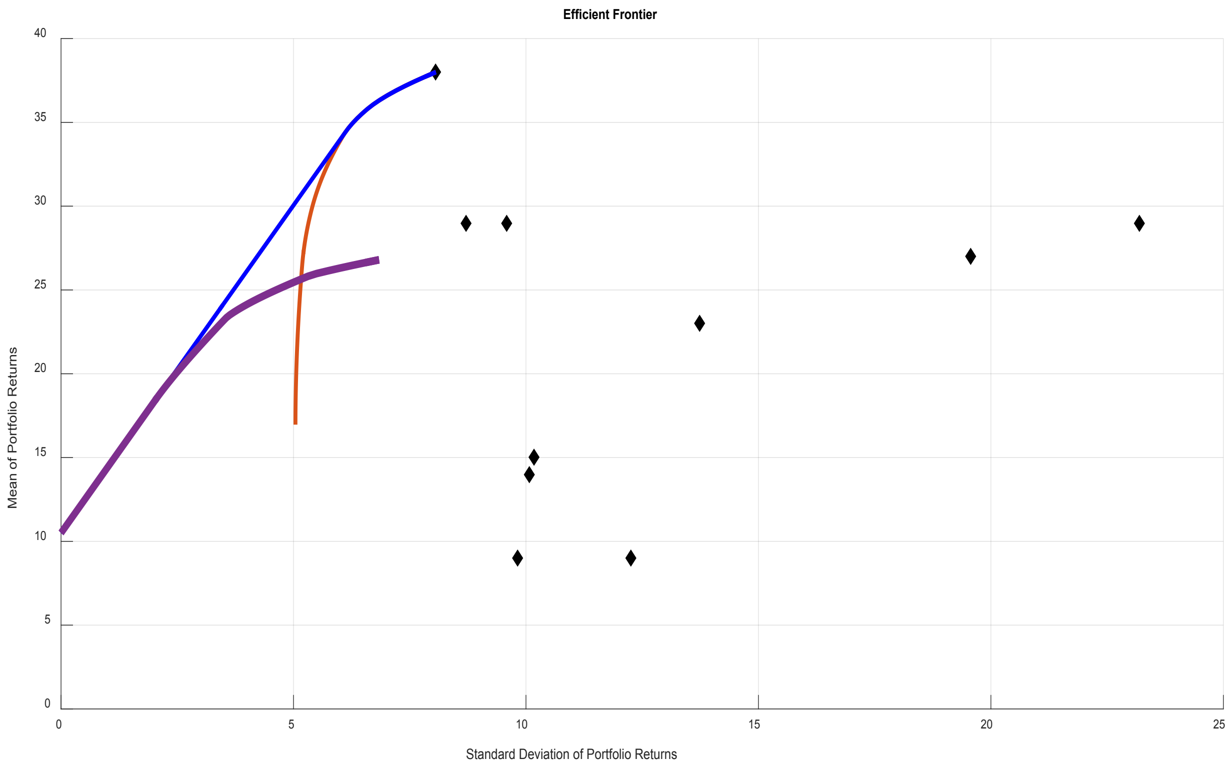

Figure 3 – The risk return trade off of the optimal portfolio with bonds and industry limits

Conclusions

The model of optimal investment portfolio was proposed which allows one to take into account the bank's objectives and risk attitude. The approach presented in this paper provides a visible and flexible methodology for designing near optimal project portfolio. An important feature of the proposed methodology is that it takes into account differences in priorities of the development bank and commercial banks. The main objective of development bank portfolio design is to determine the right combination of profitable industries, which enable the organization to achieve its expectations of the investment strategy taking into account the social and environmental issues.

The proposed methodology thus is a scientific methodology suitable for application to different data types or combination with other computing methods. Decision-makers can use the proposed method to perform analyses prior to other more precise methods.

Using the actual balance sheet and asset return data for a set of large Kazakhstan companies, this note presents several detailed examples outlining the investment selection process for a development bank manager. The presented examples explore the role of state finance institutions for affecting risk perceptions. Our analysis pays a substantial degree of attention to practical data issues and considerations which are likely to be confronted in the development bank's investment activities. Given a relatively small amount of companies considered in our examples, most of the calculations could be easily done using the classical quadratic programming methods. For larger, real world applications however, the more efficient and more robust methods, such as the Extension Optimization Method (Shukayev, 2013: 350) are of high practical value. Robust portfolio optimization aims to maximize the worst-case portfolio return given that the asset returns are allowed to vary within a prescribed uncertainty set.

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